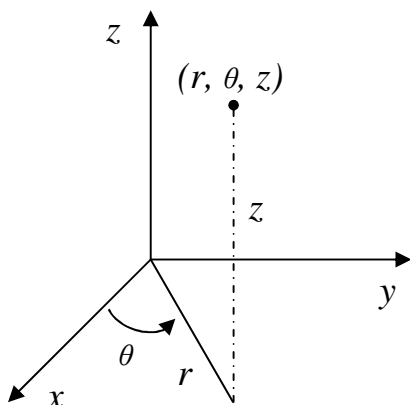
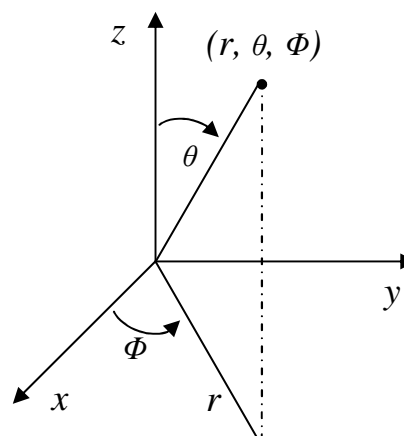


ECUACIONES DE MOVIMIENTO EN COORDENADAS RECTANGULARES, CILÍNDRICAS Y ESFÉRICAS

SISTEMAS DE COORDENADAS



Coordenadas Cilíndricas



Coordenadas Esféricas

I.- COORDENADAS RECTANGULARES

ECUACIÓN DE CONTINUIDAD

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0 \quad (1)$$

ECUACIONES DE MOVIMIENTO EN TÉRMINOS DE ESFUERZOS

Dirección x

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \rho g_x + \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \quad (2)$$

Dirección y

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \rho g_y + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) \quad (3)$$

Dirección z

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right) \quad (4)$$

ECUACIONES DE NAVIER – STOKES

Dirección x

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) \quad (5)$$

Dirección y

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) \quad (6)$$

Dirección z

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) \quad (7)$$

COMPONENTES DEL TENSOR DE ESFUERZOS VISCOSOS – FLUIDOS NEWTONIANOS

$$\tau_{xx} = 2\mu \left(\frac{\partial v_x}{\partial x} \right) + \left[\left(\kappa - \frac{2}{3}\mu \right) \nabla \cdot v \right] \quad (8)$$

$$\tau_{yy} = 2\mu \left(\frac{\partial v_y}{\partial y} \right) + \left[\left(\kappa - \frac{2}{3}\mu \right) \nabla \cdot v \right] \quad (9)$$

$$\tau_{zz} = 2\mu \left(\frac{\partial v_z}{\partial z} \right) + \left[\left(\kappa - \frac{2}{3}\mu \right) \nabla \cdot v \right] \quad (10)$$

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad (11)$$

$$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) \quad (12)$$

$$\tau_{xz} = \tau_{zx} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \quad (13)$$

DIVERGENCIA DEL CAMPO DE VELOCIDADES

$$\nabla \cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \quad (14)$$

II.- COORDENADAS CILÍNDRICAS

ECUACIÓN DE CONTINUIDAD

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0 \quad (1)$$

ECUACIONES DE MOVIMIENTO EN TÉRMINOS DE ESFUERZOS

Dirección r

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \rho g_r + \frac{1}{r} \frac{\partial(r\tau_{rr})}{\partial r} + \frac{1}{r} \frac{\partial\tau_{\theta r}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial\tau_{zr}}{\partial z} \quad (2)$$

Dirección θ

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \frac{1}{r^2} \frac{\partial(r^2\tau_{r\theta})}{\partial r} + \frac{1}{r} \frac{\partial\tau_{\theta\theta}}{\partial \theta} + \frac{\partial\tau_{z\theta}}{\partial z} \quad (3)$$

Dirección z

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \frac{1}{r} \frac{\partial(r\tau_{rz})}{\partial r} + \frac{1}{r} \frac{\partial\tau_{\theta z}}{\partial \theta} + \frac{\partial\tau_{zz}}{\partial z} \quad (4)$$

ECUACIONES DE NAVIER – STOKES

Dirección r

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rv_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] \quad (5)$$

Dirección θ

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rv_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \quad (6)$$

Dirección z

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] \quad (7)$$

COMPONENTES DEL TENSOR DE ESFUERZOS VISCOSOS – FLUIDOS NEWTONIANOS

$$\tau_{rr} = 2\mu \left(\frac{\partial v_r}{\partial r} \right) + \left[\left(\kappa - \frac{2}{3} \mu \right) \nabla \cdot v \right] \quad (8)$$

$$\tau_{\theta\theta} = 2\mu \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) + \left[\left(\kappa - \frac{2}{3} \mu \right) \nabla \cdot v \right] \quad (9)$$

$$\tau_{zz} = 2\mu \left(\frac{\partial v_z}{\partial z} \right) + \left[\left(\kappa - \frac{2}{3} \mu \right) \nabla \cdot v \right] \quad (10)$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \quad (11)$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right] \quad (12)$$

$$\tau_{zr} = \tau_{rz} = \mu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right] \quad (13)$$

DIVERGENCIA DEL CAMPO DE VELOCIDADES

$$\nabla \cdot v = \frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \quad (14)$$

III.- COORDENADAS ESFÉRICAS

ECUACIÓN DE CONTINUIDAD

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\rho v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial(\rho v_\phi)}{\partial \phi} = 0 \quad (1)$$

ECUACIONES DE MOVIMIENTO EN TÉRMINOS DE ESFUERZOS

Dirección r

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) = -\frac{\partial p}{\partial r} + \rho g_r + \left[\frac{1}{r^2} \frac{\partial(r^2 \tau_{rr})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tau_{\theta r} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r} \right] \quad (2)$$

Dirección θ

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \left[\frac{1}{r^2} \frac{\partial(r^2 \tau_{r\theta})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\tau_{\theta\theta} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi\theta}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\tau_{\phi\phi} \cot \theta}{r} \right] \quad (3)$$

Dirección Φ

$$\rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\theta v_\phi \cot \theta}{r} \right) = -\frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi} + \rho g_\phi + \left[\frac{1}{r^2} \frac{\partial(r^2 \tau_{r\phi})}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\phi}}{r} + \frac{2\tau_{\theta\phi} \cot \theta}{r} \right] \quad (4)$$

ECUACIONES DE NAVIER – STOKES

Dirección r

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu \left(\nabla^2 v_r - \frac{2v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{2v_\theta \cot \theta}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) \quad (5)$$

Dirección θ

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \mu \left(\nabla^2 v_\theta + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} \right) \quad (6)$$

Dirección Φ

$$\rho \left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\theta v_\phi \cot \theta}{r} \right) = -\frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi} + \rho g_\phi + \mu \left(\nabla^2 v_\phi - \frac{v_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi} \right) \quad (7)$$

donde:

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2}{\partial \phi^2} \right)$$

COMPONENTES DEL TENSOR DE ESFUERZOS VISCOSOS – FLUIDOS NEWTONIANOS

$$\tau_{rr} = 2\mu \left(\frac{\partial v_r}{\partial r} \right) + \left[\left(\kappa - \frac{2}{3} \mu \right) \nabla \cdot v \right] \quad (8)$$

$$\tau_{\theta\theta} = 2\mu \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) + \left[\left(\kappa - \frac{2}{3} \mu \right) \nabla \cdot v \right] \quad (9)$$

$$\tau_{\phi\phi} = 2\mu \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right) + \left[\left(\kappa - \frac{2}{3} \mu \right) \nabla \cdot v \right] \quad (10)$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \quad (11)$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = \mu \left[\frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left(\frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right] \quad (12)$$

$$\tau_{\phi r} = \tau_{r\phi} = \mu \left[\frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) \right] \quad (13)$$

DIVERGENCIA DEL CAMPO DE VELOCIDADES

$$\nabla \cdot v = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} \quad (14)$$